ON THE EFFLUENCE OF GASES WITH SOLID PARTICLES INTO VACUUM

A. L. Stasenko

UDC 532.529.5

Several viewpoints are presented concerning the process of a kinetic "freeze" during a spherically symmetric steady effluence of a gas into vacuum.

Gas streams often contain suspended solid or liquid particles which interact with it. If the stream expands without restraint as, for example, during effluence into vacuum, then the molecules may cease to collide and the flow can proceed in a free-molecules mode affecting, theoretically, the interaction between suspended particles and the gas. Such a process is conveniently analyzed in the case of a supersonic gas emitter which simulates an axially symmetric jet in the region close to the axis and which is represented in nature by a comet head.

The steady flow of a spherically symmetric gas stream with solid particles emitted from a comet nucleus (radius $r_* \sim 1$-10 km) has been analyzed in [1]. Such a flow occurs within a region of radius $r \lesssim 10^5$-10^6 km in the comet head. The equations of motion and energy for small spherical particles ($\sim 1\ \mu$m) are integrated numerically; the coefficients of resistance and heat transfer between particles and gas are assumed to depend on the gas temperature T and to determine, in the end result, the final velocity of particles at the outer boundary of that region.

The hydrodynamic parameters for the entire region of such a spherically symmetric flow are determined from the Euler equations for an inviscid and thermally nonconducting compressible fluid. The boundary between continuous and free-molecules flow modes is defined approximately by the condition that the local isotropic mean-free-path be equal to the distance from the comet center:

$$r_{\mathrm{P}} \sim e. \tag{1}$$

On the surface of a comet nucleus, $r_* = 5$ km, one assumes the following values for the concentration and the free path of molecules: $n_* \sim 10^{12}$-$10^{14}/\mathrm{cm}^{-3}$ and $l_* \sim 10$-10^3 cm, which yield the Knudsen number $\mathrm{Kn}_* = l_*/r_* = 2 \cdot (10^{-5}$-$10^{-3})$ and thus confirm the existence of a continuous medium at a comet nucleus. If molecules for which $l \sim n^{-1} \sim r^2$ are assumed rigid, then condition (1) yields $r_{\mathrm{P}} \sim 10^3$-10^5 km [1]. We will show here that this estimate is somewhat too high.

For a steady spherically symmetric supersonic stream there has been found – theoretically and experimentally – some characteristic radius r whose different lengths determined on the basis of different considerations in [2-7] are of the same order of magnitude. We will list these considerations.

1) In [2] the Crook model equation has been solved numerically for two temperature-wise "ellipsoidal" molecule velocity-distribution functions; it is shown that at $r \to \infty$ the "longitudinal" temperature $T_\parallel$ approaches asymptotically its "freeze" value, while the characteristic "transition" radius r_H of rigid molecules is determined according to the expression $(r_H/r_*)^\varkappa \sim \mathrm{Kn}_*^{-1}$. This is the radius at which the difference between longitudinal temperature $T_\parallel$ and transverse temperature $T_\perp$ becomes equal to the local gas-dynamic temperature (corresponding to the continuous-medium model): $T_\parallel - T_\perp = T$. An analogous approach has been taken in [3].

N. E. Zhukovskii Central Institute of Aerodynamics, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 22, No. 3, pp. 511-513, March 1972. Original article submitted May 28, 1971.

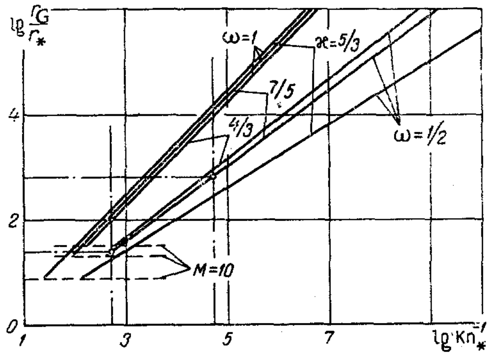

Fig. 1. "Transition" radius of a continuous stream becoming a collision-free stream, as a function of the Knudsen number at the surface of a supersonic spherical emitter.

2) In [4] the radius r_G has been determined at which the maximum difference betwen macroscopical velocities at the center and at the surface of an l-sphere becomes comparable to the mean thermal velocity of a molecule, $\sup|\mathbf{V}(\mathbf{r}_G + 1) - \mathbf{V}(\mathbf{r}_G)| = \langle c(\mathbf{r}_G)\rangle$, from where

$$\left(\frac{r_G}{r_*}\right)^{1+2(\varkappa-1)(1-\omega)} = \left(\frac{8}{\pi\varkappa}\right)^{1/2} \left(\frac{\varkappa-1}{\varkappa+1}\right)^{1+1/2(\varkappa-1)(1-\omega)} Kn_*^{-1}. \quad (2)$$

This expression is valid for a gas subject to any power-law relation between viscosity and temperature $\mu \sim T^{\omega}$ and with any constant ratio of specific heats $\varkappa$. Specifically, for rigid molecules ($\omega = 1/2$) we have then $r_G^{\varkappa} \sim Kn_*^{-1}$.

3) In [5] the anisotropic l-surfaces have been determined which "burst" (when a sample molecule ceases to collide with field molecules, at least in one direction) at a radius equal to r_G within an accuracy down to $O(M_G^{-1})$.

4) The experimental studies in [6] concerning the "freeze" of an argon stream ($\varkappa = 5/3$) have confirmed the estimates in [2], [4], and [5] of the radius corresponding to the "transition" from a continuous to a collision-free medium.

5) In [7] the "boundary" of the inviscid region in a spherically symmetric stream r_L has been estimated at which the viscous forces become comparable to the inertia forces; denoting this radius by r_G, we have $(r_L/r_G)^{1+2(\varkappa-1)(1-\omega)} = (1/2)\varkappa\pi$. According to this estimate, the Euler equations for an ideal gas become invalid approximately where the flow becomes collision-free.

Thus, from various viewpoints there can be deduced the following relation between the "transition" radius and the corresponding mean-free-path (in a concomitant system of coordinates moving with the gas):

$$\frac{l}{r_*} \sim \left(\frac{r}{r_*}\right)^{2-\varkappa} \quad (3)$$

(this relation becomes relation (1) only when l is replaced by the path length $\lambda \sim lV/\langle c\rangle$ in a fixed system of coordinates tied to the emitter).

The "transition" radii, plotted on the basis of formula (2) as a function of the Knudsen number Kn_*^{-1}, are shown in Fig. 1 for rigid molecules ($\omega = 1/2$) and for Maxwellian molecules ($\omega = 1$) on the transonic surface of an emitter. Shown are also the constant-M (Mach number) lines $r_M/r_* = M^{1/(\varkappa-1)} \times [(\varkappa+1)/(\varkappa-1)]^{-(\varkappa+1)/4(\varkappa-1)}$ for M = 10, according to formula (2) derived in the supersonic approximation $V/a_* = [(\varkappa+1)/(\varkappa-1)]^{1/2} + O(M^{-2})$. It can be seen here that, at $Kn_*^{-1} = 5 \cdot (10^2 - 10^4)$, $\varkappa = 7/5$, $\omega = 1/2$, and $r_* = 5$ km in [1], the estimate for the "transition" radius is $r_G \sim 5.25$ to 5.700, i.e., $\sim 10^2 - 10^3$ km rather than $r_P \sim 10^3 - 10^5$ km according to formula (1). Thus, the stream "freezes" much sooner and solid macroscopic particles carried by this stream will, at $r > r_G$, interact with the hotter gas ($T_G > T_P$), which will somewhat affect the accelerating force as well as the thermal flux imparted to a particle. As an end result, the final velocity of macroscopic particles leaving the comet head will change (perhaps only slightly).

The author's aim here was to again point out that for engineering-physics purposes it is necessary, in principle, to account for the "freeze" effect where molecules are subject to translatory degrees of freedom in steady and freely expanding gas streams, or to estimate the error incurred by disregarding this effect in calculations.

LITERATURE CITED

1. R. F. Probstein, in: Problems in Hydrodynamics and Mechanics of Continua [in Russian], Nauka (1969).
2. B. B. Hamel and D. R. Willis, Phys. Fluids, 9, No. 5 (1966).
3. G. A. Luk'yanov and V. A. Silant'ev, Izv. Akad. Nauk SSSR Mekhan. Zhidk. i Gaza, No. 5 (1968).
4. A. L. Stasenko, Inzh. Fiz. Zh., 16, No. 1 (1969).
5. A. L. Stasenko, Inzh. Fiz. Zh., 18, No. 4 (1970).

6. J. B. Anderson and J. B. Fenn, Phys. Fluids, 8, No. 5 (1965).
7. M. D. Ladyzhenskii, "Supersonic flow of gas in space," Mashinostroenie (1968), Prikl. Matem. i Mekhan., 26, No. 2 (1962).